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# View factors for circular air ducts in attics: fast and accurate approximations

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**View factors for circular air ducts in attics: Fast and accurate approximations**

by

**Hiep Tran**

A thesis submitted to the graduate faculty  
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Mechanical Engineering

Program of Study Committee:  
Baskar Ganapathysubramanian, Major Professor  
Travis Sippel  
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Iowa State University

Ames, Iowa

2016

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**DEDICATION**

This thesis work is dedicated to my parents, Ngoc Kieu and Hoang Tran and to my sister Anh Tran who have been a wonderful source of support and encouragement during the challenges of graduate school and life. You always have loved me unconditionally and your living examples have taught me to work hard for the things that I aspire to achieve. I am truly thankful for having you in my life.

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**ABSTRACT**

Attic energy modeling is a topic of interest as current whole building energy models have a difficult time predicting the thermal performance of attics. Recent developments in attic energy modeling has allowed, for the first time, many different attic shapes to be analyzed. This allows analysis of roofs beyond standard gable, shed, flat, and saltbox type roofs. With this generalization of attic geometries, the calculation of view factors becomes a potential issue for these diffusely emitting nonconvex polygon shaped attics. The presence of air ducts further complicate calculation of view factors and radiation heat transfer in attics. Currently there are two methods to produce these view factors, 1) solving the double integral that defines the view factor or 2) simple calculations based on engineering assumptions to allow for the view factors to be easily calculated. The first approach can be computationally intensive as air ducts and complex geometry can drastically increase the number of surfaces to be calculated in the integral equation, and can be tedious to set up. On the other hand, the second (very fast) approach is built upon assumptions in ASHRAE RP-717. Very limited analysis has been documented on the validity of these assumptions. Furthermore, with current advances in the ability to model different attic geometries these assumptions may no longer be valid. This work focuses on updating the view factor assumptions to provide a fast and accurate method for calculating view factors in attics containing circular air ducts under more generalized conditions. The assumptions are updated by first systematically evaluating the old assumptions to determine whether a modification is needed, followed by designing and testing an alternative approach when needed. In all cases, extensive evaluation and comparison of proposed alternative approach with the older assumptions and double integral calculations are performed. Finally, a comparison between approach in RP-717 and the new rules developed in this thesis is reported. We envision that these updated assumptions

can easily be included in whole building or attic specific energy modeling software to better capture the thermal effects of radiation heat transfer between air ducts and attic surfaces.

## CHAPTER 1: INTRODUCTION

### 1.1 Background

Space heating and cooling consume roughly 54% of energy in residential buildings[1]. The building envelope of a residential building serves as a thermal barrier and has a crucial role in affecting interior temperatures and the amount of energy needed to maintain thermal comfort. Attics are a part of the building envelopes, and can be a significant contributor to the energy consumption of a residential building. In fact, the building envelope accounts for 56% of heating and cooling loads while attic spaces and roofs are responsible for 12% - 14% [1]. Good attic space designs and proper insulations help reduce heating and cooling loads in the building, thus resulting in energy savings for homeowners. For examples, in southern climate homeowners can install radiant barriers in an attic space, which can save approximately \$150 per year [2].

Compared to the rest of the building envelopes, attic spaces are very unique. Attics often experience high ventilation rates, large temperature swings throughout the day, experience high radiative loads, and may contain air ducts, HVAC systems, skylights, and windows. Each of these features create problems for standard whole building energy simulation platforms [3] as most of the algorithms are design specifically for conditioned spaces. Of these aspects, of particular interest in this work, is the presence of air ducts in attics and their influence on thermal performance of attics.

Although it is recommended to place air ducts in the conditioned space [4], there are several reasons to install HVAC and air ducts in non-conditioned space (such as attics or garages) [4]. Air ducts may need to be relocated because of interior designs or space requirements during a retrofit. If a building has historical significance, then modifications to the envelope or interior may need to

be limited. Homeowners, architects, engineers, and building professionals in different climate zones around the country have to consider those variables in order to have proper air duct systems that best fit their needs and provide efficient heating and cooling loads in the building. Air ducts, being part of the HVAC systems, have a significant contribution to the energy consumption of residential buildings. Before cooling or heating air reaches conditioned spaces from the central HVAC system, up to 30-40% of the thermal energy can be lost along air ducts due to conduction [4]. This loss cuts down the HVAC efficiency by up to 18% [5]. Air ducts are also responsible for 12% up to 30% of air leakage area of residential buildings [6], [7]. Air leakage and conduction losses result in high energy bills for homeowners, increase peak demand for utilities, and degrades the apparent HVAC performance. Duct leakage is also another major issue for homeowners. Air ducts leakage can decrease indoor air quality such as introducing polluted air from the surrounding into living spaces (especially if there is moisture problems in ventilated attics) [8]. Thus the presence of air ducts is an important factor towards the energy efficiency of a residential building.

## **1.2 Objectives**

The overall goal of this thesis is to improve the modeling of radiative heat transfer for air ducts in attics by developing a fast and accurate algorithm for view factor calculations. This goal is achieved by improving the current assumptions and methods used in ASHRAE RP-717 [3] – which is the standard protocol for estimating the view factors in attics, but is more than three decades old. These assumptions are evaluated using a set of base configurations. Black body and diffusely emitting view factor matrices are computed using a numerical framework View3D [9],

which numerically computes the double integral defining the view factor. If the assumption in RP-717 performs poorly, then an alternative method is proposed. The alternative method is then evaluated with the same base cases used to evaluate the methods of RP-717, such that a direct comparison can be made. After each of the assumptions has been determined to be sufficient or been updated, the total view factor matrices are computed and compared with the matrices produced by View3D. This research aims to improve the assumptions made in RP-717 and produce an easy to use method that can be eventually implemented in whole building and attic specific energy modeling software.

### **1.3 Scope**

Chapter 2 provides a literature review on different tools and standards used to investigate the energy performance of air ducts and attic designs in residential building. Chapter 3 describes the heat balance equations of an attic space, view factor concepts for ducts sitting on an attic floor, displays different attic geometries, duct configurations, verifies the numerical duct representation, provides summary of the base configurations, and the approaches used to verify duct view factor assumptions. Chapter 4 presents the results of verification process for each assumption and shows alternative approach to each inaccurate assumption. Chapter 5 demonstrates the accuracy improvements the new methodology compared to the RP-717 method. Finally, conclusions and recommendations are presented in Chapter 6.

## CHAPTER 2: LITURATURE REVIEW

Different organizations have created tools and standards to investigate the energy performance of attics and air duct designs in residential building. A model presented in the ASHRAE fundamentals is a steady state duct model useful for sizing air duct systems [10]. In RP-717 there are several duct models described that are all based on the principle of the U-value of the ductwork and an energy balance on differential length of a duct [3]. The U-value model includes heat capacity of air with heat conduction through thermal transmittance along the length of the duct. The ASHRAE SP43 model incorporates transient effects and temperature changes along multiple ducts [11], [12]. Modera developed an air leakage steady state model along a differential length of an air duct [13]. Modera's model is simplified to the U-value model if there is no air leakage along a duct run [13]. These models do not include radiation exchanges between surfaces in the presence of ducts. Parker et al. also showed steady state duct model in attics that has some features such as heat radiation between roof deck and external surface of duct and simple model for cycle of HVAC system [14]. The view factor between the air duct and the roof decks in this model assumes a two dimensional spaces. However, as shown later in this thesis, view factors usually need to be based on three dimensional spaces. Ober and Wilkes developed a three dimensional cylindrical air duct model which had all fundamental aspects of ASHRAE SP43, combined air leakage features from Modera's model, and simple cycle aspects of HVAC developed by Parker et al. [3]. This model is included as part of ASHRAE Research Project RP-717 [3] and ASTM standard practice C1340 (AtticSim) [15]. However, the model described in RP-717 has some drawbacks, namely that ducts only block radiation from the attic floor and that view factor calculations are limited to specific geometries such as gable ended, flat, saltbox, and shed attic

geometries. The view factors of ducts in attic space are calculated based on a set of simplified assumptions [3]. Furthermore, the assumptions (which are listed below) have limited documentation of their performance.

Assumptions made in RP 717:

**Assumption 1:** For radiation heat transfer calculations, each duct may be treated as being isothermal, gray, and diffusely emitting and reflecting.

**Assumption 2:** View factor between any duct run and the floor of the attic enclosure is 0.5.

**Assumption 3:** The presence of ducts reduces the view factors from the attic floor to the other attic surfaces by a constant factor.

**Assumption 4:** The view factor from a duct to a particular attic surface is the same for all duct segments.

**Assumption 5:** The presence of ducts modifies only those view factors that involve the attic floor.

**Assumption 6:** The ducts are small enough that view factors between any two duct runs may be considered negligible compared with view factors between the ducts and the attic surfaces.

Other than using engineering assumptions and analytical formulas for calculation of the view factor matrix, a numerical software that computes view factors based on solving the integral equation can be used. There is currently a software, View3D [9], used to evaluate view factors. The framework was written in C originally by Walton at NIST [16]. Gaussian quadrature is used to evaluate the view factor double integral. The framework can solve view factors for both convex and non-convex geometries and use adaptive refinement to solve obstructed view factors. View3D also has built-in functionality with the ability to calculate diffusely emitting view factors [17].

Other view factor investigations for circular or cylindrical objects have been investigated in the recent past. Ameri and Felske [18] investigated view factors at various angles of two circular cylinders that have the same radius with different lengths and distances from either their connecting ends or around joining centers. However, it provides limited information about the effect of various radiuses on view factors. Juul [19] provided analytical expressions used to evaluate view factors between two parallel oriented cylinders of same finite length and different radius.

Although these numerical and analytical tools allow for the calculation of view factors, each approach has specific disadvantages. Introducing circular ducts in View3D can be fairly computationally intensive compared to using analytical and engineering assumptions. Since the circular duct needs to be discretized into a set of surfaces around the cylindrical object, the number of surfaces in the attic geometry increases rapidly. For example, a simple gable ended attic has 5 surfaces that bound the attic space (2 roof decks, 2 gables, and an attic floor), and (as shown in chapter 3) to properly represent a duct the cylindrical object should be discretized into approximately 16 surfaces. The geometry of the attic may be difficult or tedious to create with the View3D program as the vertices and surface connectivity information is needed for each surface. As for analytical solutions, their major limitation is that only limited cases are available and not all air duct layouts in three dimensional space are covered by the analytical solutions. This chapter has discussed the current state-of-the-art methods for calculating view factors for cylindrical objects and air ducts in attics, while the next chapter shows how the view factors are introduced into the governing equations of air ducts and attic surfaces.

## CHAPTER 3: METHODS

This chapter shows the general heat balance equations of attic spaces, a differential duct sitting on the attic floor, and view factor concepts. The chapter also shows numerical verification of the duct representation, a summary of the base configurations, and the approaches used to verify duct view factor assumptions.

### 3.1 Governing Equations

Before talking about assumptions of view factors of duct inside an attic enclosure, it is necessary to discuss the general energy balance equations of attic spaces with air ducts, figure 1. The heat balance energy equations of the attic model described in RP-717 is based on equations originally developed by B. Peavy [20] and extended by Wilkes [20], [21]. The heat balance equation of inside attic surfaces, eq. 1, includes conduction through surface, radiation between surfaces, convection between surface and attic air, moisture at surface, radiation between duct and surface and radiation from trusses:

$$Q_i(i) + Q_{ri}(i) + Q_{ci}(i) + Q_{moist}(i) + Q_{rD}(i) + Q_{rt}(i) = 0 \quad \text{for } i = 1:N \quad (1)$$

The heat balance equation of outside attic surfaces, eq. 2, includes conduction through surface, radiation from outside surfaces to surroundings, convection between surface and air solar radiation on surface:

$$Q_o(i) + Q_{ro}(i) + Q_{co}(i) + \alpha(i)Q_s(i) = 0 \quad \text{for } i = 1:N \quad (2)$$

The heat balance for the attic air includes convection from all the surfaces, air ducts and trusses, ventilation and infiltration, and air leakage from the ducts.

$$\sum_{i=1}^{n_s} A(i)q_{ci} + Q_{\text{vent}} + Q_{cD} + Q_{ct} + Q_L = 0 \quad (3)$$

For air ducts, the duct is assumed to be symmetric along the centerline of the duct. Using this assumption, a heat balance is performed on the outside surface, eq. 4, the inside surface, eq. 5, and the air duct air temperature, eq. 6.

$$Q_{o,d}(i) + Q_{co,d}(i) + Q_{r,d}(i) = 0 \quad \text{for } i = 1:N_d \quad (4)$$

$$Q_{i,d}(i) + Q_{ci,d}(i) = 0 \quad \text{for } i = 1:N_d \quad (5)$$

$$Q_{\text{HVAC}_{in}}(i) + Q_{ci,d}(i) + Q_{\text{leak}}(i) + Q_{\text{HVAC}_{out}}(i) = 0 \quad \text{for } i = 1:N_d \quad (6)$$

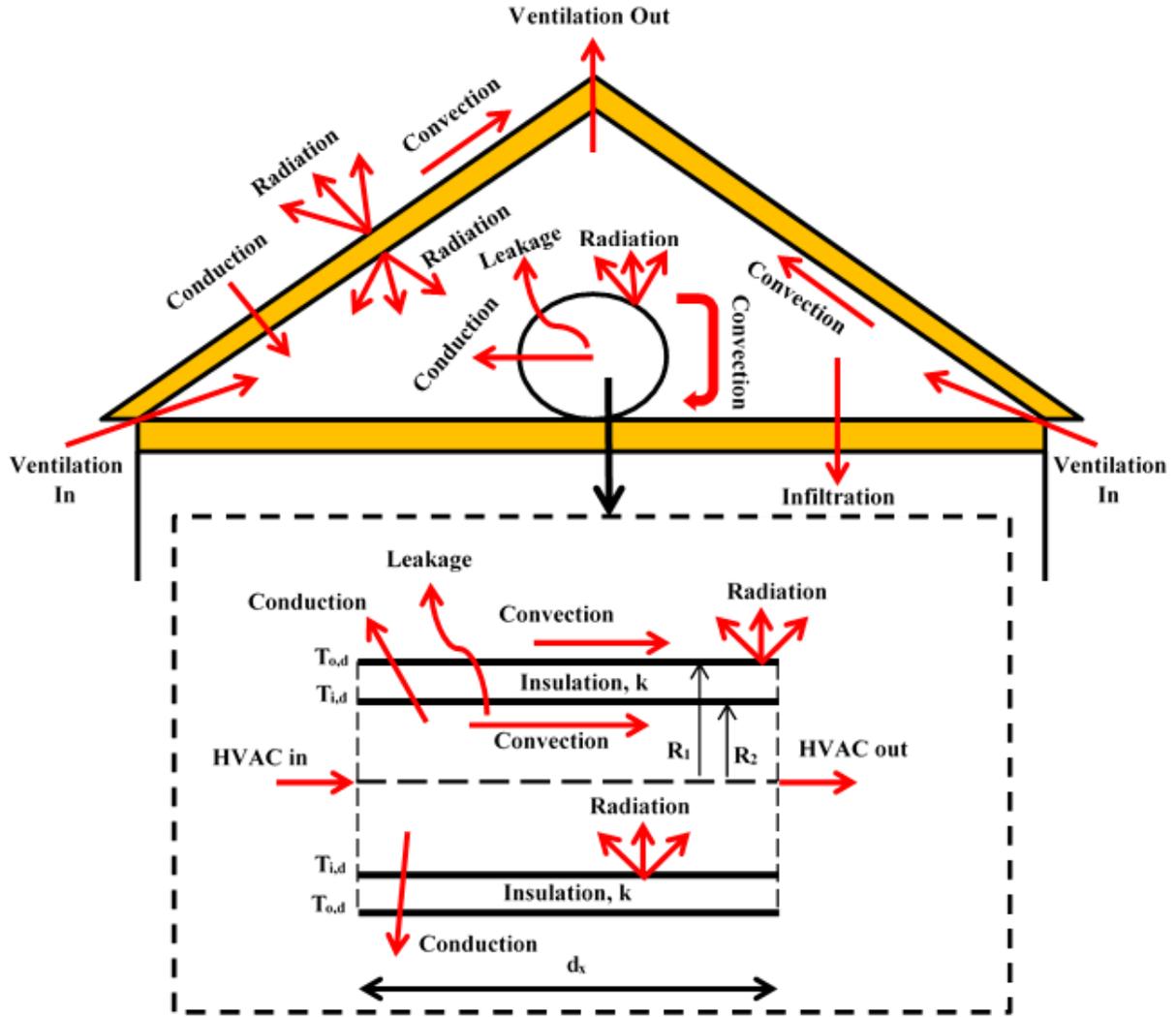
The radiative component on the outside surface of the differential duct segment,  $Q_{r,d}$ , contains the view factors,  $F_{d \rightarrow j}$ , from each duct segment to other surfaces in the attic, eq. 7.

$$Q_{r,d}(i) = \sum_{j=1}^N \varepsilon_{d,i} \sigma F_{d,i \rightarrow j} A_d(i) (T_{d,i}^4 - T_j^4) + \sum_{j=1}^{N_d} \varepsilon_{d,i} \sigma F_{d,i \rightarrow d,j} A_d(i) (T_{d,i}^4 - T_{d,j}^4) \quad (7)$$

for  $i = 1:N_d$

The other radiation term on the inside surfaces in eq. 1 of the attic also effects the radiation being emitted by the air ducts, eq. 8.

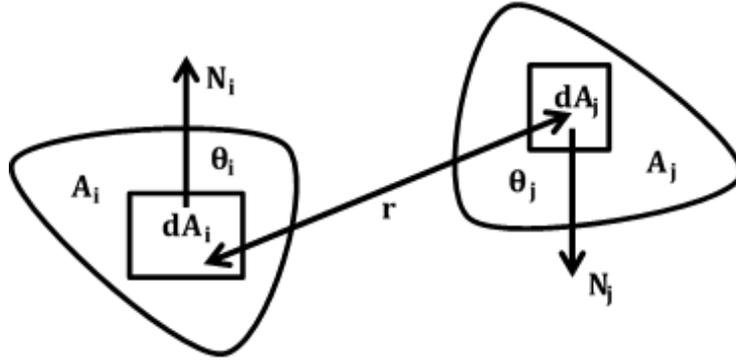
$$Q_{rD} = \sum_{j=1}^{N_d} \varepsilon_i \sigma F_{i \rightarrow d,j} A(i) (T_i^4 - T_{d,j}^4) \quad \text{for } i = 1:N \quad (8)$$



**Figure 1. Different modes of heat transfer of an attic space in the presence of duct (top) & (bottom) different mode of heat transfer in differential length of a duct.**

The view factor calculation method then needs to calculate the view factor between any  $i^{\text{th}}$  duct to any  $j^{\text{th}}$  surface,  $F_{d,i \rightarrow j}$ , any surface  $i$  to any duct  $j$ ,  $F_{i \rightarrow d,j}$ , and radiation between any two attic envelop surfaces  $F_{i \rightarrow j}$ . The view factor between any two arbitrary surfaces is defined as double area integral [17], eq. 9 and is illustrated in figure 2.

$$F_{i \rightarrow j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} dA_i dA_j \quad (9)$$



**Figure 2. Illustration of symbols used in double area integration of view factor equation**

The view factors also must satisfy the reciprocity formula [17], eq. 10,

$$A_i F_{i \rightarrow j} = A_j F_{j \rightarrow i} \quad (10)$$

and in an enclosure, the sum of the view factors from a given surface  $i$  to all surfaces in the enclosure must be 1.0, eq. 11.

$$\sum_{j=1}^N F_{i \rightarrow j} = 1 \quad (11)$$

Overall the view factors create a square matrix,  $\mathbf{F} \in \mathbb{R}^{(N+N_d) \times (N+N_d)}$ , with a row sum of 1.0 from the enclosure equation, eq. 11. This view factor matrix can be calculated by View3D, which integrates equation 9 by Gaussian quadrature and accounts for obstructions in non-convex geometries and can adjust the black body view factors to diffusely emitting view factors<sup>1</sup>.

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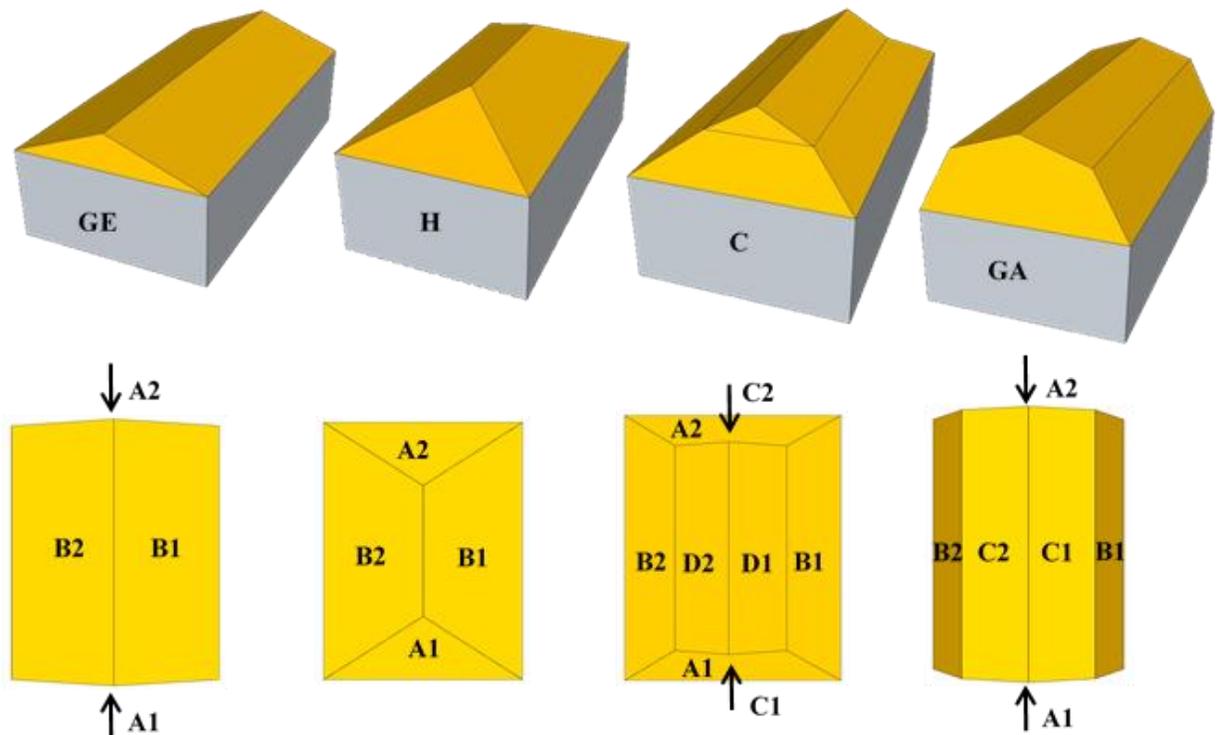
<sup>1</sup> For more information about how View3D calculates these view factors and the calculation of the diffusely emitting view factors, please see appendix A.

### 3.2 Description of Attic & Duct Base Configurations

The section introduces the base attic and duct configurations that are designed to evaluate the assumptions in RP-717.

#### *Description of attic configurations*

Previous research works accepted by ASTM and ASHRAE only consider gable ended, flat, shed and saltbox roofs as standard attic structures for commercial and residential buildings. Using the new attic shape generalization of the Fraunhofer Attic Thermal Model (FATM) [15] a few different attic shapes are selected: gable-ended roof, hip roof, combination roof and gambrel roof, figure 3. These geometries use a roof that has been previously analyzed by RP-717 (the gable end), a non-convex attic geometry (combination roof), and a few other relatively common attic shapes (hip and gambrel).



**Figure 3. Attics shapes (top - left to right): gable-ended (GE), hip (H), combination (C) & gambrel (GA). Top views (bottom - left to right) of each shape.**

The different attic configurations used in this research are constructed based on ASTM C1340 [15]. The angle between each roof and the horizontal plane of each attic enclosure is 22.62 degrees. The base dimensions are 28' x 55' which remains constant throughout all attic configurations. Besides configuring attic structures as black body, components of an attic structure also have their own emittance factors to represent common materials used to construct an attic space. All roofs of attics represent oriented strand board (OSB) that has emittance factor of 0.91 [22] and all attic floors represent fiberglass that has emittance factor of 0.75 [23]. All chosen attic shapes, dimensions and emittance factors make the research practical while same base dimensions provide good mutual comparisons later.

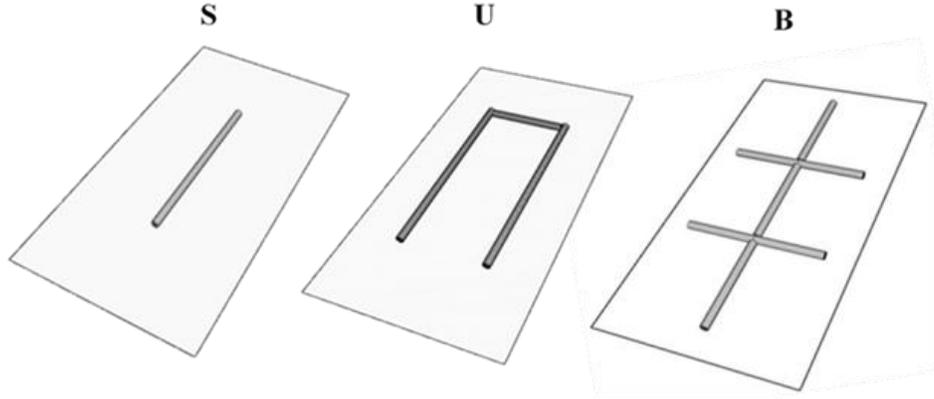
### ***Description of duct configurations***

Besides having various attic shapes such as gable-ended, hip, combination and gambrel roofs, different duct configurations are also used in this work. Three different duct configurations are chosen: single duct, u-shape duct, and branch duct, figure 4. Circular cross section of duct is based on ASTM C1340 standards [15]. In addition, duct shapes are chosen similar to regular air ducts system seen in common residential or light commercial buildings. A duct outer diameter used in this research for all three configurations is 1.16 ft. which is based on the ASTM C1340 standard [15]<sup>2</sup>. The location of each duct configuration is positioned on the attic floor and is centered in the middle of the attic. Each duct configuration also has two different emittance factors besides black body, galvanized iron (0.9) and polished aluminum (0.05). They are common materials used in manufacturing air duct. All selected duct shapes, emittance factors and outer

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<sup>2</sup> Other duct diameters might be used based on the specific assumption that is being evaluated.

diameters are all based on typical configurations in residential buildings, materials and sizes. Combining them with different attic configurations discussed previously create various base configurations to verify view factors of duct assumptions in an attic space.

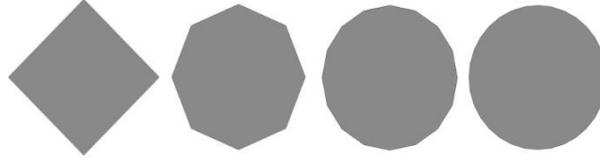


**Figure 4. Duct configurations (from left to right): single (S), u-shape (U) & branch (B). All duct configurations all sit on the attic floor.**

### 3.3 Numerical duct representation

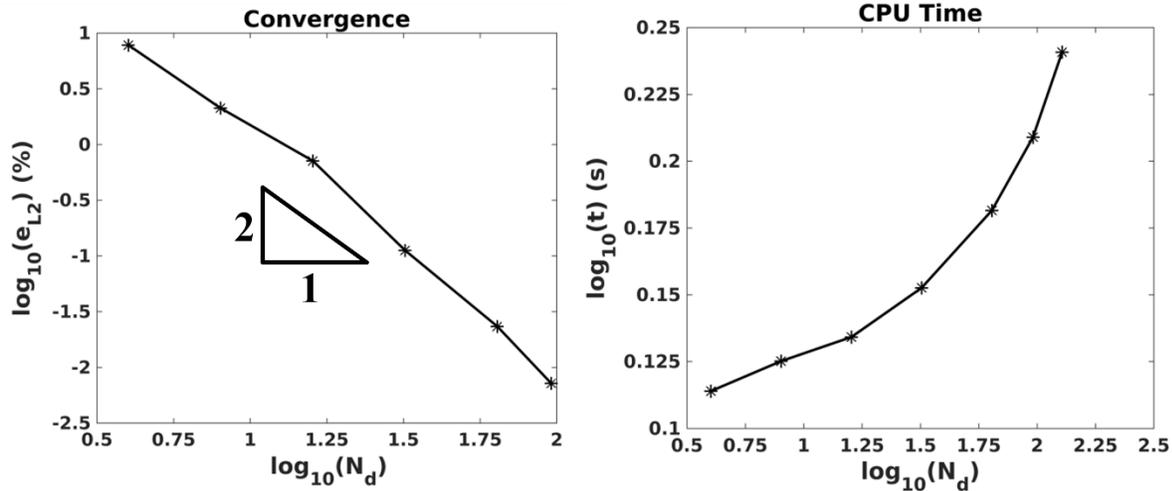
The base cases are first analyzed with the view factor software View3D to provide numerical approximations of the view factor matrices for future comparison and evaluation of the engineering assumptions. In order to ensure that the duct is spatially resolved with a sufficient number of surfaces, the duct is discretized with increasing set of surfaces -- 4, 8, 16, 32, 64, 96, and 128 surfaces, figure 5. These different duct radial resolutions were simulated with View3D in the single duct and the gable end configuration. The L2 relative error norm, eq. 12, was used to compare different radial resolutions with the finest radial resolution.

$$e_{L2} [\%] = \sqrt{\frac{\sum_{i=1}^N \sum_{j=1}^N \left( F_{i \rightarrow j}^{(\text{coarse})} - F_{i \rightarrow j}^{(\text{fine})} \right)^2}{\sum_{i=1}^N \sum_{j=1}^N \left( F_{i \rightarrow j}^{(\text{fine})} \right)^2}} \times 100 \quad (12)$$



**Figure 5. A cross-section view of circular air duct discretized by set of quadrilateral shaped surfaces. The figure (from left to right) shows air duct constructed of 4, 8, 16 and 32. The cross-section view of duct that is discretized by 64, 96 and 128 is similar to one constructed by 32 surfaces.**

Figure 6 shows the spatial convergence of the view factor matrix relative to the finest discretization. The solution is converging at a rate of roughly 2.0, and drops below 1% at 16 radial surfaces. From this point forward, 16 surfaces will represent the ducts when View3D is used to calculate the view factors. Although more surfaces result in a more accurate view factors, the computational time increases with the number of surfaces used to radially approximate the cylindrical duct, figure 6. Overall the time might not seem too large. However, if many duct runs are simulated or the problem involves plenum duct system with many branches from the main plenum section, the simulation will take a substantial amount of simulation time. Due to this limitation, it is necessary to use a simple but accurate duct representation to approximate view factors of ducts in attics whenever possible.



**Figure 6. Convergence of the relative  $L_2$  errors as the discretization numbers of duct surface increase (Left) & the CPU simulation time required to calculate view factor by using View3D (Right).**

### 3.4 Summary of base configurations cases used to verify duct view factor assumptions

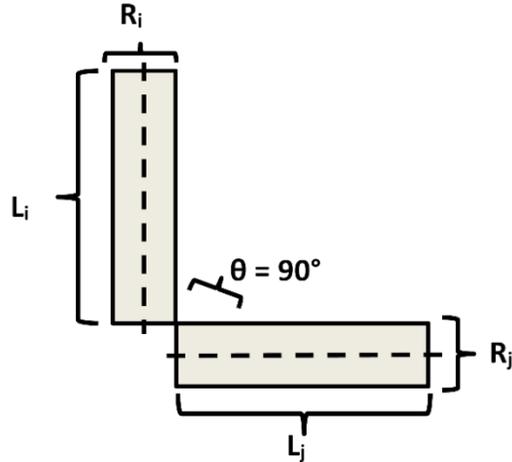
As discussed previously, four different attic geometries (such as gable-ended, hip, combination, and gambrel), three duct shapes (such as single, u-shape and branch), duct outer diameters, 16 surfaces used to represent duct, and various emittance factors are selected as inputs for the View3D simulations. Table 1, shows the different set-up cases used to evaluate each assumption from RP-717.

There is no need to verify the first assumption, for radiation heat transfer calculations, each duct may be treated as being isothermal, gray, and diffusely emitting and reflecting. This is because in reality attic roofs are made out of different materials that have various emittance factors. During simulation process, View3D also initially treats surfaces of an enclosure as black body and then use black body view factors later in post processing to produce grey diffusely emitting view factors.

**Table 1. Attic, duct configurations and emittance factors used to verify duct view factor assumptions.**

<b>Assumptions</b>	<b>Attic Configuration</b>	<b>Duct Configuration</b>
2. The view factor between any duct run and the floor of the attic enclosure is 0.5.	All selected attics. Emittance factors: 1. Black body 2. All roofs are OSB & floor is fiberglass	All selected duct shapes Emittance factors: Black body Galvanized Iron Polished Aluminum
3. The presence of ducts reduces the view factors from the attic floor to the other attic surfaces by a constant factor.	1. Gable-ended 2. Combination Emittance factors: Black body 2. All roofs are OSB & floor is fiberglass	1. Single duct (S) 2. Branch duct (B) Emittance factors: Black body 2. Galvanized Iron 3. Polished Aluminum
4. The view factor from a duct to a particular attic surface is the same for all duct segments.	1. Gable-ended Emittance factors: Black body 2. All roofs are OSB & floor is fiberglass	1. U-shape duct (U) Emittance factors: 1. Black body 2. Galvanized Iron 3. Polished Aluminum
5. The presence of ducts modifies only those view factors that involve the attic floor.	All selected attics Emittance factors: Black body	All selected duct shapes Emittance factors: 1. Black body
6. The ducts are small enough that view factors between any two duct runs may be considered negligible compared with view factors between the ducts and the attic surfaces	Set-ups used to verify this assumption is different from the rest and described briefly later in this section.	

The set-up cases for the last assumption is different from the rest. The enclosure geometry can be any shape since the only interested variable in this case is view factor between two ducts. A duct configuration similar to previous works developed by Ameri et al [18] is used to verify this assumptions along with some changes. The configuration is two perpendicular ducts (Figure 7). Additional details about the approach used to verify this assumption are discussed in the next section



**Figure 7. Two perpendicular ducts connected at one end.**

### **3.5 Approaches used to verify each duct view factor assumptions**

In this section, View3D is briefly introduced, and some of the RP-717 methods for calculating duct view factors in attic enclosures are described.

#### ***A brief introduction of View3D framework***

As discussed previously in the introduction, View3D software can solve view factors for both convex and non-convex geometries. View3D first reads through a geometry text file. Then the framework uses Gaussian quadrature and adaptive integration in order to solve view factors between surfaces. It has the capability to solve for obstructed, unobstructed view factors of black body and diffusely emitting surfaces. Finally, the framework outputs a text file that contains surface view factors of an enclosure in form of matrix and their surface areas. More detailed information about View3D program is in Appendix A.

#### ***Assumption 2: View factor between any duct run and the floor of the attic enclosure is 0.5***

This assumption which states view factor between any duct and the attic floor is 0.5 only applicable in black body case. The view factor will be changed after taking emittance factors into consideration. The view factors of all surfaces in an enclosure will be treated as black body first

and then black body view factor results are used in a post processing step to produce new view factors that include emittance factors. For the base configurations that include emittance factors, two cases are simulated by View3D in order to evaluate the view factor between a duct and attic floor, figure 8. The attic and duct configurations used in two simulation runs are gable-ended (GE) and single duct (S) with different emittance factors. These duct and attic configurations are the same as model in RP 717.



**Figure 8. Two simulation runs used to evaluate diffusely emit view factors between duct and floor by View3D. The results from those two simulation runs will be used as a benchmark for other set-ups that involve similar emittance factors as shown in table**

**Table 2. Four attic shapes combined with a duct configuration along with three different sets of emittance factors such as black body for all surfaces; roofs are made out of oriented strand board (OSB); attic floor is made out of fiberglass and duct is made out of oriented strand board (OSB); attic floor is made out of fiberglass and duct is made out of either galvanized iron or aluminum.**

GE	S	Roof, Floor, Duct: black body
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Duct: galvanized iron (0.9)
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Duct: polished aluminum (0.05)
H	U	Roofs, Floor, Ducts: black body
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Ducts: galvanized iron (0.9)
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Ducts: polished aluminum (0.05)
GA	B	Roofs, Floor, Ducts: black body
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Ducts: galvanized iron (0.9)
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Ducts: polished aluminum (0.05)

**Assumption 3: The presence of ducts reduces the view factors from the attic floor to the other attic surfaces by a constant factor.**

In order to find the blocking factor in the presence of ducts in an attic enclosure, a simple algorithm developed by Ober and Wilkes [3] is used, eq. 13-16.

$$\sum_{j=1}^N F_{f \rightarrow j} + \sum_{k=N+1}^{N+N_d} F_{f \rightarrow k} = 1 \quad (13)$$

Sum of view factors from attic floor to attic surfaces and from attic floor to ducts in an attic is one. This is similar to eq. 13 which states summation of all view factors from one surface to the rest in an enclosure is equal to 1. As discussed in previous assumption 2, the view factor from any duct to an attic floor is 0.5. Additional evaluations are done for two emittance cases used in single duct (S) and gable-end (GE) configurations beside black body: 1) Attic roofs are OSB, attic floor is fiberglass and ducts are galvanized iron, and 2) Attic roofs are OSB, attic floor is fiberglass and ducts are polished aluminum. Three view factors of black body, case 1 and 2 are reused in this assumption again to calculate view factor from floor to ducts in attic. They are evaluated by using the reciprocity formula, eq. 14, then the analytical blocking factor can be found, eq. 15.

$$A_f F_{f \rightarrow d} = A_d F_{d \rightarrow f} \quad (14)$$

$$B = 1 - \sum_{k=1}^{N+N_d} F_{f \rightarrow k} \quad (15)$$

The blocking factor (B) is the difference between the total view factors of a floor to other surfaces in an attic enclosure. Finally, the view factor from the floor to a particular attic surface in the presence of ducts is calculated, eq. 16.

$$F'_{f \rightarrow j} = B * F_{f \rightarrow j} \text{ for } j = 2, 3, 4 \dots, N \quad (16)$$

In this formula, view factors from floor to attic surfaces without ducts  $F_{f \rightarrow j}$  are calculated by using View3D software. Then, the product of  $F_{f \rightarrow j}$  and B, eq. 16 is the view factor between the floor and an attic surface in the presence of duct. The results obtained this approach are plotted along with the simulation approach which directly calculate  $F'_{f \rightarrow j}$ .

Duct and attic configurations used in the evaluation process are single duct (S), branch duct (B), gable-ended (GE) and combination (C). Using convex attic GE and non-convex attic C are sufficient to verify the assumption. This is because the hip roof (H) and gambrel (GA) are also same geometry types and provide similar information. The duct configuration u-shape (U) is not used in the verification process because the branch duct (B) provides adequate information to make judgement on the assumption.

***Assumption 4: The view factor from a duct to a particular attic surface is the same for all duct segments.***

A simple method is used to determine the view factor from any duct segment to a particular attic surface according to the assumption. Gable-ended (GE) and hip (H) both have four attic surfaces. Combination (C) and gambrel (GA) have eight attic surfaces. Those surfaces do not include attic floor. Black body view factors between a duct and an attic floor are determined previously in assumption 2. They are used in eq. 17 to calculate the black body view factors of each duct segment and an attic surface. Post processing steps which are described in Appendix A are also implemented to calculate diffuse view factors.

$$F_{d \rightarrow f} + \sum_{j=1}^{N-1} \beta = 1 \quad (17)$$

The view factors between a duct segment and attic floor are evaluated previously in different emittance configurations: 1) Black body, 2) Attic roofs are OSB, attic floor is fiberglass and ducts are galvanized iron 3) Attic roofs are OSB, attic floor is fiberglass and ducts are polished aluminum. Table 3 shows duct view factors of different emittance base configurations for attics with 4 non-attic floor surfaces. The same calculation procedure can be applied to other attic

surfaces such as combination (C) and gambrel (GA) that have 8 non-attic-floor surfaces. These view factors from a duct to attic surfaces are compared against View3D simulation results.

**Table 3. View factors of any duct segments to a particular surface in three emittance set-ups for 4 roofs.**

Number of non-attic floor surfaces	Black body	Roofs: OSB (0.91) Floor: fiberglass (0.75) Duct: galvanized iron (0.9)	Roofs: OSB (0.91) Floor: fiberglass (0.75) Duct: polished aluminum (0.05)
4	0.125	0.125	0.0075

Other attics provide the same information that does not affect final outcomes. Duct configurations single duct (S) and branch duct (B) are not used in the verification process because single duct (S) provides the view factors from only itself to other attic surfaces. The branch duct (B) has seven duct segments and provides good information as the u-shape duct (U) configuration. The U-shape duct (U) simplifies the simulation process and provides adequate information to make judgment on the assumption.

***Assumption 5: The presence of ducts modifies only those view factors that involve the attic floor. For example, it is assumed that the ducts do not change the view factor between the two roof surfaces nor the view factors between the two gables nor the view factors between either of the roof surfaces and either of the gables.***

The assumption does not specify how small the ducts can be such that the ducts obstruct large portions of the surfaces in an attic. In the cases where ducts occupy a large quantity of the attic, the ducts do significantly obstruct the view factors for all the attic surfaces. As the size of the ducts are reduced the effect of the ducts reduce as well. Hence, it is important to determine some measure that shows when the ducts no longer significantly obstruct the view factors. Since view factors are based on the size of two different surfaces, an area ratio between the air duct total

area and the total attic surface area is chosen,  $\frac{A_d}{A_a}$ . If the ratio is big, it means total surface area of ducts is small compared to the attic. As a result, the presence of ducts does not change view factors among surfaces in an enclosure and ducts can be neglected in an attic. In contrast, if the total surface area of ducts is small, the ducts may alter the view factors between the attic surfaces. All attics and duct configurations are used as set-up configurations to determine the desired ratio of surface areas, so that the assumption can be accurately implemented. Only black body emittance factor is considered, because the View3D program use black body results in post processing steps in order to produce new view factors that include emittance factors.

A simple problem is solved in order to determine the ratio of surface area between an attic enclosure and ducts in all set-up configurations. The relative matrix  $e_{L2}$ , eq. 18, is computed between view factor matrix  $F_{i \rightarrow j}$  of an attic with duct configurations at various diameters (4, 2, 0.5, 1.16 and 1/12 ft.) and a similar attic enclosure but empty. The relative matrix shows overall effect of different duct diameters in blocking views among surfaces in an enclosure. In addition, absolute matrix  $e_{L\infty}$ , eq. 19 is also calculated in a similar process. The absolute matrix shows the effect of duct diameters on individual view factors among surfaces of attic.

$$e_{L2} [\%] = \sqrt{\frac{\sum_{i=1}^N \sum_{j=1}^N \left( F_{i \rightarrow j}^{(d)} - F_{i \rightarrow j}^{(\text{empty})} \right)^2}{\sum_{i=1}^N \sum_{j=1}^N \left( F_{i \rightarrow j}^{(\text{empty})} \right)^2}} \times 100 \quad (18)$$

$$e_{L\infty} [\%] = \text{Max} \left| \sum_{i=1}^N \sum_{j=1}^N \left( F_{i \rightarrow j}^{(d)} - F_{i \rightarrow j}^{(\text{empty})} \right) \right| \times 100 \quad (19)$$

***Assumption 6: The ducts are small enough that view factors between any two duct runs may be considered negligible compared with view factors between the ducts and the attic surfaces.***

The assumption does not specify exactly the range in which ducts are small enough so that their view factors might be negligible. As discussed previously, figure 7 is used in this work to investigate the range in which view factors of two ducts might be negligible. The duct layout is similar to general configuration of duct segments in light commercial buildings. Since Ameri et al [18] only considered two ducts that have the same length and radius, extra modifications are taken into account such various radiuses and lengths for both ducts. Having different radii and lengths help expanding the view factor evaluation process between two ducts in an enclosure and provide useful information about their view factors. This section addresses the view factor between two perpendicular cylindrical ducts with different lengths and radii.

The view factor between two ducts in this case depends on four different variables, such as  $L_1$ ,  $L_2$ ,  $R_1$  and  $R_2$ . The parameters can be normalized into 3 parameters  $(\frac{R_1}{L_1}, \frac{R_2}{L_1}, \frac{L_2}{L_1})$  describing the view factors between two perpendicular ducts, with the parameters. Simulations of the view factors along each of these axes produce a volumetric function representation of the duct view factors. The non-dimensional ranges considered in this section can be seen in table 4. These ranges are discretized logarithmically with 21 points in the range for a total of 9261 points that evaluate the volumetric view factor function. The approach can easily be extended to ducts with acute or obtuse angles. A similar approach can also be used for parallel ducts.

**Table 4. Simulation values associated with each variable.**

$\theta$ (degree)	$\frac{R_1}{L_1}$	$\frac{R_2}{L_1}$	$\frac{L_2}{L_1}$
90	$[\frac{1}{200}, 1]$	$[\frac{1}{200}, 1]$	$[\frac{1}{50}, 50]$

## CHAPTER 4: EVALUATION AND MODIFICATION OF ASSUMPTIONS

All results obtained from View3D simulations and simplified approaches are shown in this chapter. Alternative methods to an assumption are discussed if needed, and an overall conclusion for each verification process is shown.

### **4.1 Assumption 2: View factor between any duct run and the floor of the attic enclosure is 0.5**

#### *Evaluation Results*

The view factors from a duct to an attic floor in three base configurations involving black body and emittance factors are shown. For the black body case, the RP-717 assumptions result for a view factor from a duct segment to the attic floor is 0.5. For the emittance configurations, the attic surfaces are constructed of oriented strand board (OSB) for the roof decks, the attic floor is fiberglass, and the duct is galvanized iron (emittance = 0.9) for the first case and polished aluminum (emittance = 0.05) for the second case. The case with the duct being constructed of galvanized iron on duct, the RP-717 assumptions result in a view factor from duct to attic floor of 0.37. For the case with the duct being constructed with polished aluminum, the RP-717 assumptions result in a view factor from duct to attic floor of 0.02. The results of view factor from any duct to an attic floor for a single duct, u-shape duct and branch duct configurations in all attics are shown in figure 4<sup>3</sup>.

The results overall show that for the black body case, View3D calculates the view factors to be between 0.485 - 0.49. This range is very close to the assumption, 0.5. For two emittance

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<sup>3</sup> Notice that in figure 9, Black means black body for everything; Fe means galvanized iron on duct, OSB on roofs and fiberglass on attic floor; and Al means aluminum on duct, OSB on roofs and fiberglass on attic floor

cases, the diffuse view factor varies from 0.35 to 0.36 for galvanized iron duct and from 0.018 to 0.019 for polished aluminum duct. The ranges are not that different from the RP-717 assumption values of 0.37 and 0.02. The error over all from simulation results to the assumption view factors ranges from 1% to 3%. Error below 5% is acceptable for most applications.

View factors between a duct and an attic floor in single duct (S) configurations are very close to the assumption. This is expected because there is no blockage between duct surfaces and attic floor. However, there are some obstructions for u-shape duct (U) and branch duct (B) configurations. For u-shape duct (U) set-ups, the two ducts 1 and 3 have same view factors since they are symmetric and also see the floor better than duct 2 due to larger surface areas. In contrast, duct 2 cannot see the whole floor due to blockage from the adjacent ducts. Similarly, all ducts cannot see the whole floor completely in the branch duct (B) configurations. Each duct only sees a portion of the floor because of blockage created by other ducts. The view factors of duct 2 and 3 are the same due to symmetry. Likewise, a group of ducts 1, 4 and 7 and pair of ducts 5 and 6 also experience the same effect. As a result, the duct blockage has a larger effect in high emittance cases, but in low emittance cases adjacent ducts do not greatly affect the view factor between the duct and the floor.

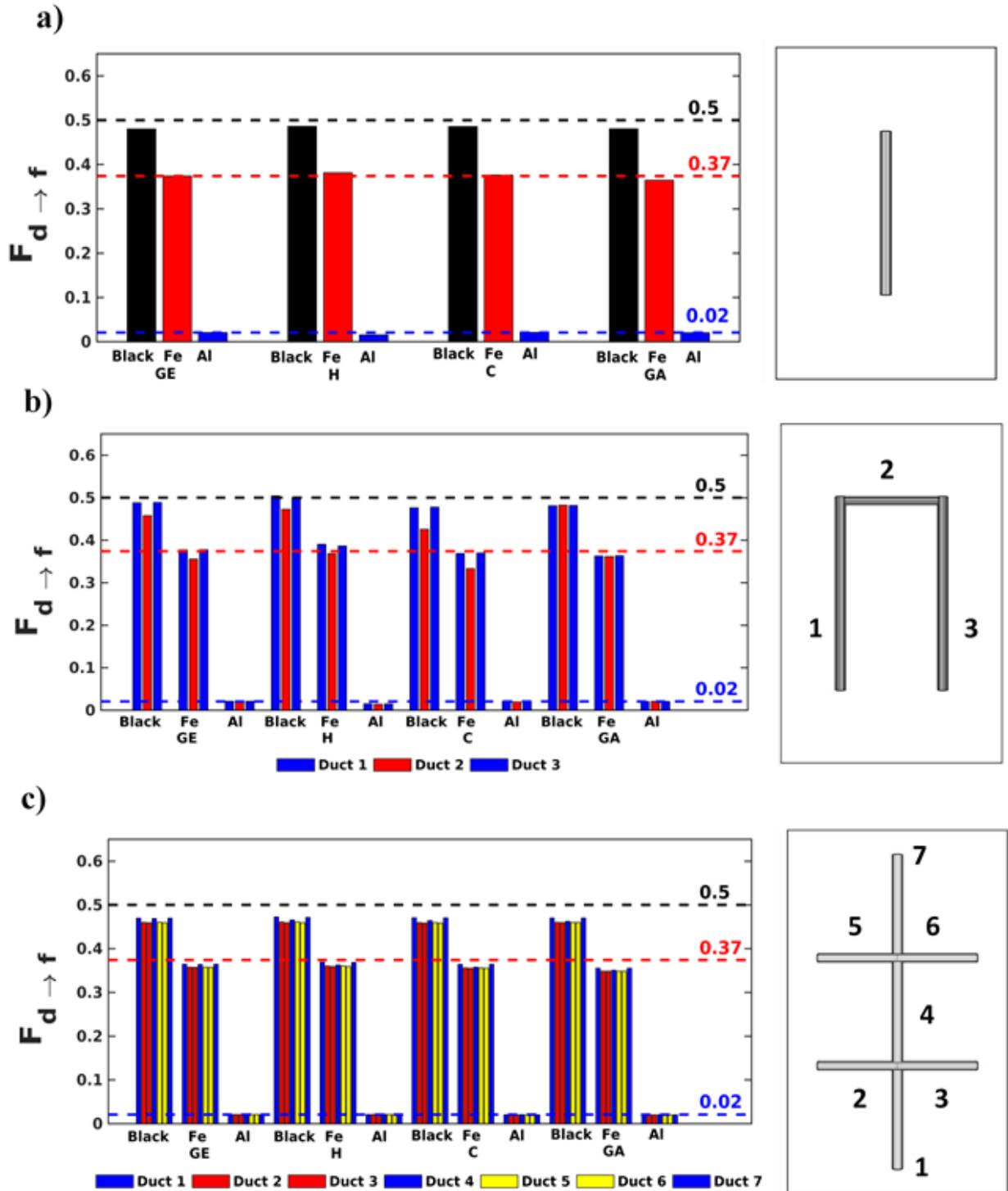


Figure 9. Different set-ups used to verify assumption 2. From top to bottom: a) Single-duct (S), b) U-shape duct (U) and c) Branch duct (B) in different attic geometries and emittance factor.

### ***Result of Evaluation***

Overall this is a good assumption used to evaluate view factor between air duct and attic floor in attic enclosures. Overall the errors are less than 5% which is considered acceptable. This assumption works best in case air ducts are made out of materials that have low emittance factors under the single duct runs where other ducts are not obstructing the view factor between the duct and the floor. Due to the performance of this assumption an alternative approach is not needed.

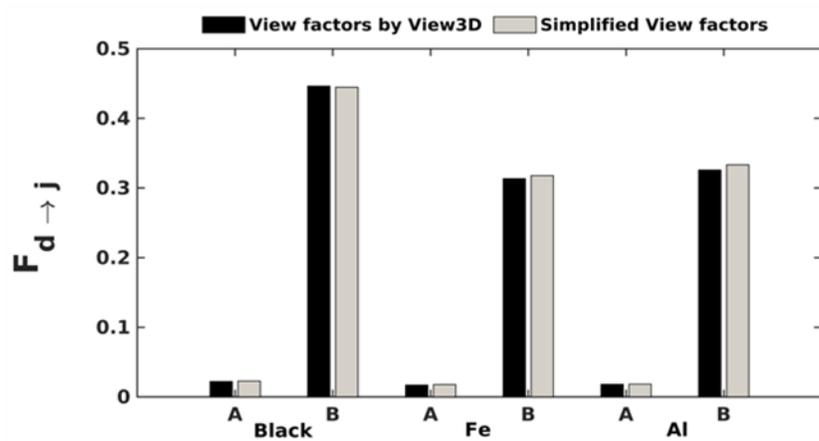
### **4.2 Assumption 3: The presence of ducts reduces the view factors from the attic floor to the other attic surfaces by a constant factor**

#### ***Evaluation Results***

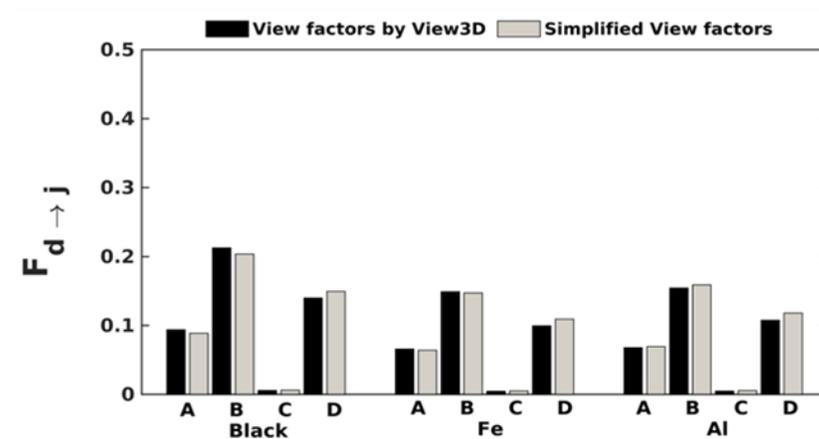
The view factors from an attic floor to different surfaces in attic enclosure of different configurations are shown in figure 10. Figure 10 shows results obtained by View3D software and results based on the RP-717 assumptions. The results overall show that view factors computed by the assumption and View3D are approximately the same. The error between results that are calculated by both methods ranges from 1% to 3%. View factors from the floor to attic surfaces in the presence of single duct (S) and branch duct (B) show the same trend for gable-ended (GE) and Combination (C) roofs. The floor sees both surfaces B better than surfaces A because surfaces B have larger surface areas. Symmetries are expected in two plots of single duct (S) and branch duct (B) due to similarity of surfaces in attic enclosure. In the combination roof configurations, the view factors are generally lower compared to that in gable-ended (GE) cases. This is simply because the combination roof configuration has more surfaces. The view factor between the attic floor and the C surfaces is small, because they are perpendicular to the attic floor and the area of the C surfaces

are small. Similarly, to single duct (S) set-up, there are also symmetries in view factor from the floor to other attic surfaces as well in branch duct (B) set-up.

a)



b)



**Figure 10. View factor from floor to attic surfaces in different set-ups. From top to bottom: a) Single-duct (S) in gable-ended (GA) and b) Branch duct (B) in combination (C) with various emittance factors.**

### *Result of Evaluation*

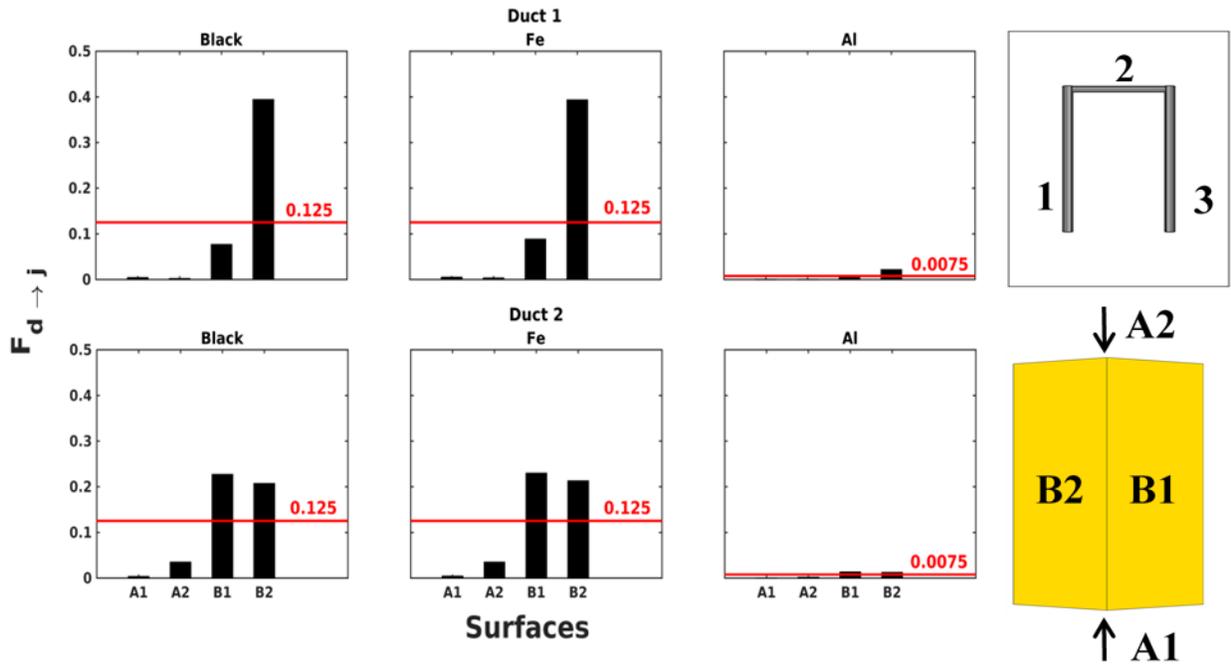
Based on the results for the two different configurations, the RP-717 assumptions are the approximately the same as View3D simulations. Overall the errors are less than 5 %. This is a

good assumption in case the attic has one zone and all ducts are installed in that zone. Thus, view factors between attic floor and other attic surfaces can be evaluated correctly. In cases the attic has multiple zones and ducts are located only in one zone, this assumption might not work well. This is because ducts only block view of the floor in the zone where they are installed. Separating zones where ducts are installed from the rest is a way to apply this assumption. Based on the performance of the assumption, an alternative approach is not needed for single zone simple floor plan attics.

#### **4.3 Assumption 4: The view factor from a duct to a particular attic surface is the same for all duct segments**

##### ***Evaluation Results***

The view factors from any duct segment to different surfaces in attic enclosure of different configurations are shown in figure 11. figure 11 shows results obtained by View3D software and results based on the assumption. The results overall show that view factors computed based on the assumption and View3D simulations are not similar. The assumption assumes that any duct segments have the same view of the attic surfaces. However, this is not true, because the view factor between any duct segment and attic surfaces depend on location of duct in the attic space, attic geometry, and surface areas of the surface and the duct. Figure 11 shows view factors of duct 1 and 2 of the u-shape duct (U) to different attic surfaces in the gable-ended (GE) attic. Duct 1 can see surface B better than any surfaces because of the duct location. This trend is also applied in the two other emittance configurations. Due to the low emittance of aluminum duct 1 and 2 have very low diffuse view factors between all the surfaces.



**Figure 11. View factors from two duct segments to attic surfaces in gable-ended (GE) setups. From top to bottom: a) duct 1 of the U-branch duct shape and b) duct 2 of the U-branch duct shape along with various emittance factors.**

### *Result of Evaluation*

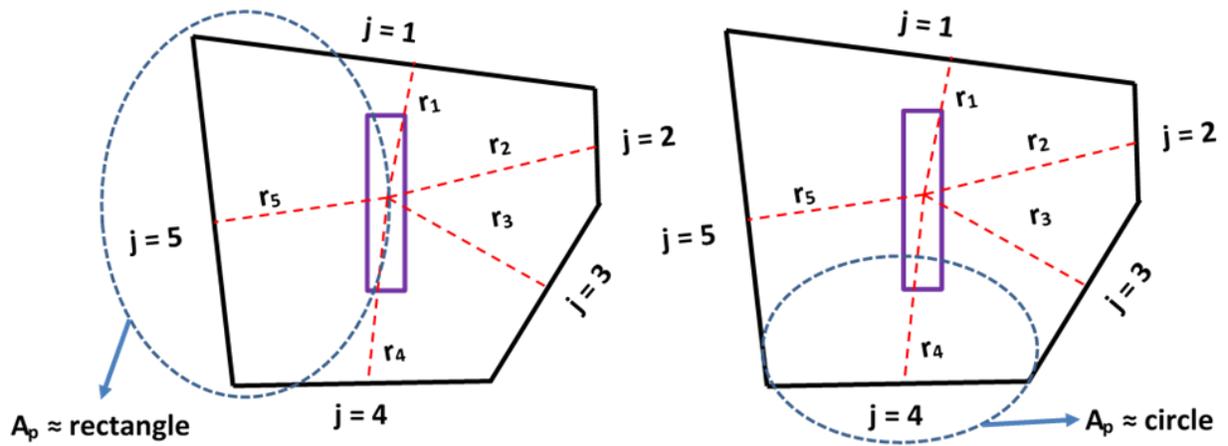
Based on figure 11 of duct 1 and 2, the view factor results calculated based on assumption are not correct for high emittance factor cases. Ducts at various locations on the floor view attic surfaces differently. Hence, the assumption cannot be used to compute view factors from any duct segments to attic surfaces. However, when ducts use low emissivity materials such as aluminum, the view factors between ducts and attic surfaces are very low. As a result, view factors most likely do not contribute significantly to the overall radiation heat transfer of ducts in an attic space and the assumption might be useful in these cases. Based on this performance an alternative method is needed especially in the high emittance cases.

### Alternative Method

Instead of having just a constant factor  $\beta$  eq. 17, a weighting factor needs to be developed to include the distance from the duct to a given surface and how well a duct can see a given surface. Using this information, the weighting factor is constructed with two ratios. The first ratio in eq. 20 is a ratio of the average centroid distance between the duct and all the surfaces, eq. 21, and the distance to the specific surface where the view factor is being evaluated.

$$F_{d \rightarrow f} + \sum_{j=1}^{N-1} \beta \left[ \frac{(\bar{r})^2}{(r_j)^2} \times \frac{(A_p + A_j)}{(A_s)} \right] = 1 \quad (20)$$

$$\bar{r} = \frac{1}{N-1} \sum_{j=1}^{N-1} r_j \quad (21)$$

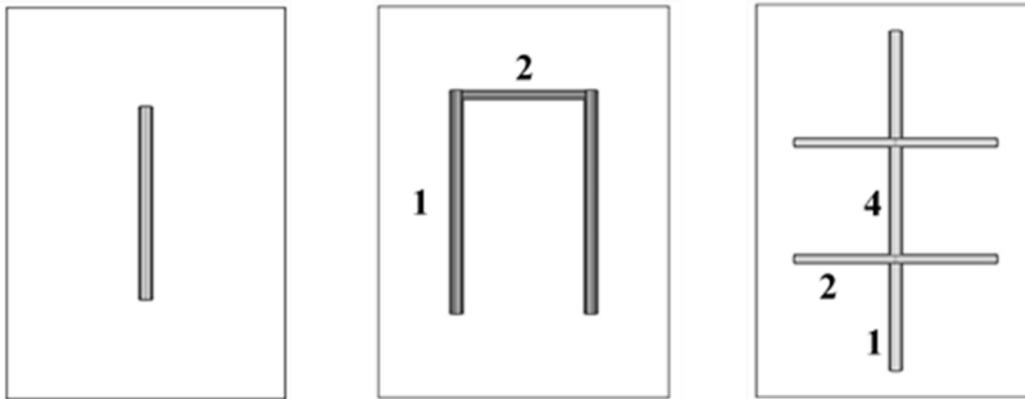


**Figure 12. Illustration of distance from a duct to each surface  $j$  and projection area of a duct to each surface  $j$  in an enclosure (from left to right, top view).**

The second ratio weights the sum of the the projected area of a duct on surface  $j$  and area of surface  $j$  to the total area of roofs. In order to simplify the process of obtaining the projected areas of duct on each surface  $j$ , the projected areas can be either circle or rectangle with respect to the view

direction of duct to surface  $j$ . If the enclosure is non-convex, the duct might not be able to see some surfaces. Hence, the projection areas of the duct to those surfaces are zeros. The weight should make the view factor evaluation process between a particular duct segment and an enclosure surface better than the original assumption. This is because the weight takes into account distance between a duct and enclosure surfaces and the projected areas of ducts onto the enclosure surface. In eq. 20, everything is known except for the normalizing constant  $\beta$ . This constant enforces the enclosure criteria based on the values of the weighting function. After solving for  $\beta$ , the specific view factor between a duct and a given surface  $j$  can be calculated by eq. 22.

$$F_{d \rightarrow j} = \beta \left[ \frac{(\bar{r})^2}{(r_j)^2} \times \frac{(A_p + A_j)}{(A_s)} \right] \quad (22)$$



**Figure 13. Duct configurations and segments (from left to right): single (S), u-shape (U) & branch (B). Duct segments used in this section have numbers next to them.**

The view factors between different duct segments and the bounding surfaces of the attic are compared with the different configurations in figure 13. Eq. 20 and eq. 22 are applied in all attic and duct configurations as shown in figure 3 and figure 4. Black body emittance is considered only in this section. Black body view factors are always computed first before they are used as inputs in post processing steps which are described in Appendix A to produce diffuse view factors.

Due to symmetry only some duct segments in each configuration are used. These segments are sufficient enough to provide information about view factor between them and surfaces in an enclosure. Duct segments are labeled with numbers as shown in figure 13. Duct outer diameter for all segments is 1.16 ft. and circular cross section of duct used here which are similar to one in ASTM C1340 [15].

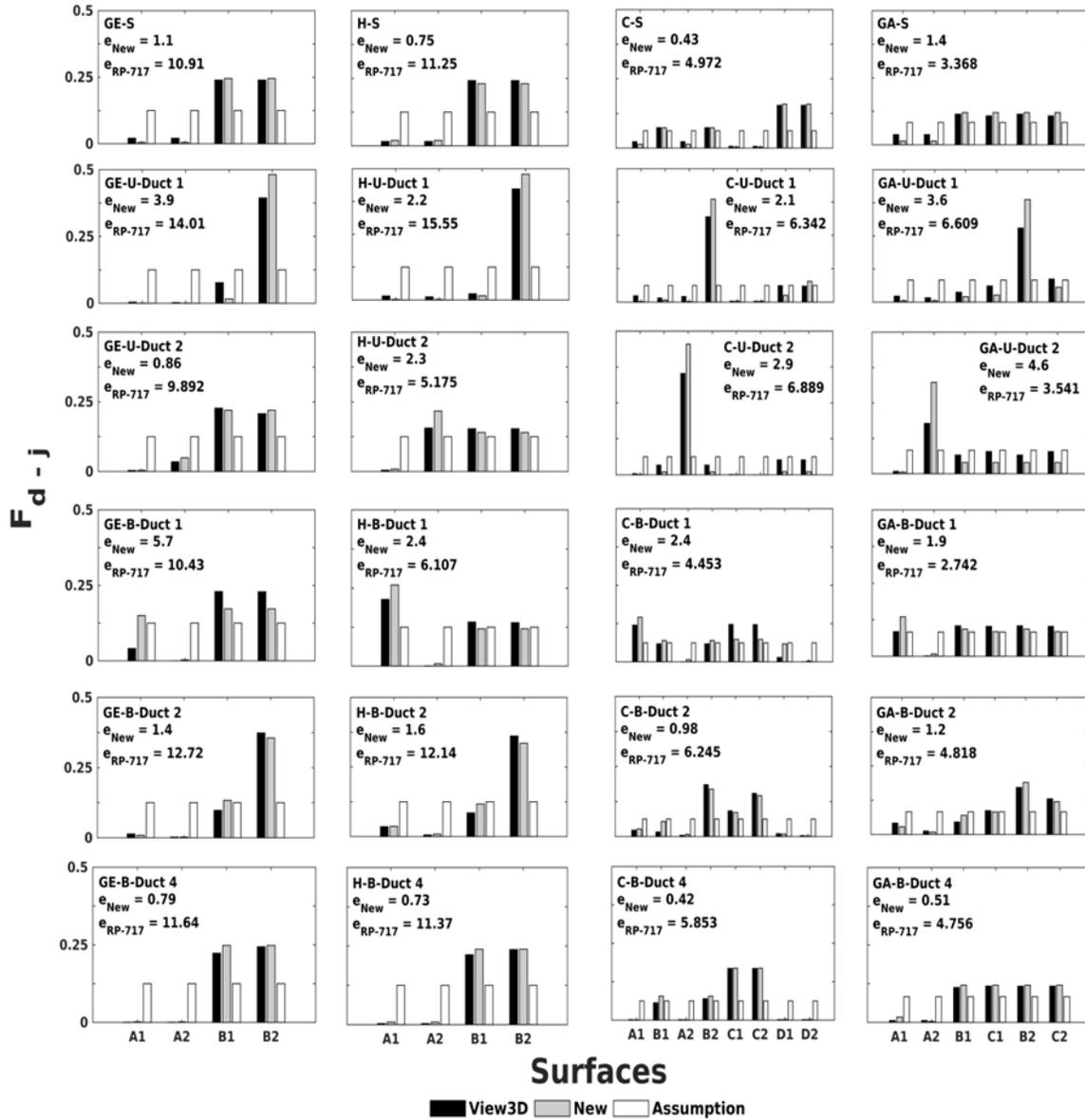
All plots are shown in figure 14 for each duct and attic configuration. Each contains the view factors calculated by three different methods such as View3D, new method described here and RP-717. Two simple error calculations are done in order to see how close between new method and View3D eq. 23 as well as between RP-717 and View3D eq. 24. Each error for each case is also shown in figure 14 respectively.

$$e_{\text{New}} [\%] = \frac{\sum_{j=1}^{N-1} |F_{d \rightarrow j, \text{New}} - F_{d \rightarrow j, \text{VIEW3D}}|}{N - 1} \times 100 \quad (23)$$

$$e_{\text{RP-717}} [\%] = \frac{\sum_{j=1}^{N-1} |F_{d \rightarrow j, \text{RP-717}} - F_{d \rightarrow j, \text{VIEW3D}}|}{N - 1} \times 100 \quad (24)$$

### ***Evaluation Summary of Alternative Method Results***

The overall results show that view factors computed by new method are better than RP-717 method, figure 14. The new method produces view factors are close to results obtained by View3D in most cases. While RP-717 method shows each duct segment has the same view factors to all attic surfaces, the new method indicates each duct segment at different location on attic floor view attic surfaces differently due to distance between them and area ratios. The  $e_{\text{New}}$  is smaller than  $e_{\text{RP-717}}$  and under 5% in most cases. This range is considered acceptable for most applications.



**Figure 14. View factors from each duct segment to attic surfaces and errors associate with each case.**

In addition, it signals that new method is useful in computing view factor between any duct segments and attic surfaces.

### ***Conclusion of Alternative Method Results***

Alternative method produces better view factors between any duct segments and attic surfaces. This new method is better than the RP-717 method and provides results close to the

View3D simulations. There are some cases in which view factors obtained by new method and View3D are not close to each other. This might due to incorrect projection areas of ducts to surface j.

**4.4 Assumption 5: The presence of ducts modifies only those view factors that involve the attic floor. For example, it is assumed that the ducts do not change the view factor between the two roof surfaces nor the view factors between the two gables nor the view factors between either of the roof surfaces and either of the gables.**

### *Evaluation Results*

For the evaluation for this RP-717 assumption a range of duct sizes was analyzed to determine when the ducts significantly affect the view factors between the bounding attic envelope surfaces. Figure 15 shows the results for the different base configurations. The relative ( $L_2$ ) and absolute ( $L_\infty$ ) errors overall drop below 5% when ratio of surface area between attic space and duct configuration is bigger than 15. Errors below 5% are acceptable for most applications. Hence, it is a good idea to neglect the duct configurations inside an attic space. This is because ducts are small enough that it will not obstruct any views among attic surfaces. In the plots of u-shape duct (U) of hip (H), combination (C) and gambrel (GA) attics, the convergence slopes are smaller than that of gable-ended (GE). This might due to duct location on the attic floor, number and orientations of attic surfaces. The errors are above 5% in all plots when ratio area between attic space and duct configuration less than 15. The bigger total surface area of duct configuration, the smaller the ratio is. Hence, duct configuration significantly blocks views among surfaces in an enclosure. As a result, duct representation in an attic can no longer be neglected and the assumption is not valid to use.

### ***Result of Evaluation***

The assumption is valid when the ratio of surface area between attic and duct configurations is bigger than 15. In this situation, the duct representation inside an attic can be ignored since it does not provide any obstructions among surfaces. However, the assumption is no longer valid when the ratio of surface area is less than 15 since duct configuration is relatively large and it obstruct surfaces in an enclosure. The efficient way to evaluate the view factors between surfaces in an attic enclosure when duct configuration is relatively big is to use View3D software.

**4.5 Assumption 6: The ducts are small enough that view factors between any two duct runs may be considered negligible compared with view factors between the ducts and the attic surfaces.**

### ***Evaluation Results***

A range of three dimensionless parameters  $\frac{R_1}{L_1}, \frac{R_2}{L_1}, \frac{L_2}{L_1}$  was analyzed to determine when the view factors are small that they can be negligible. Figure 16 shows the isosurfaces that represent different level of view factors according to ranges of three dimensionless parameters. The view factors between two ducts is below 0.05 when  $\frac{R_2}{L_1} \in \left[ \frac{1}{200}, 0.5 \right], \frac{R_1}{L_1} \in \left[ \frac{1}{200}, 1 \right]$  and  $\frac{L_2}{L_1} \in \left[ \frac{1}{50}, 50 \right]$ . Neglecting the radiation between two ducts may be acceptable in most applications when the view factor is below 0.05. Hence, a simplifying assumption may be applied in the recommended interval above to neglect duct to duct radiation. This is because their view factors are small enough that it will not affect the total view factor between each duct and attic surfaces. When the air ducts are at acute angles, this may not be a good assumption and needs further investigation. As a result, the view factor evaluation process in attic can be simplified for ducts at perpendicular or obtuse angles.

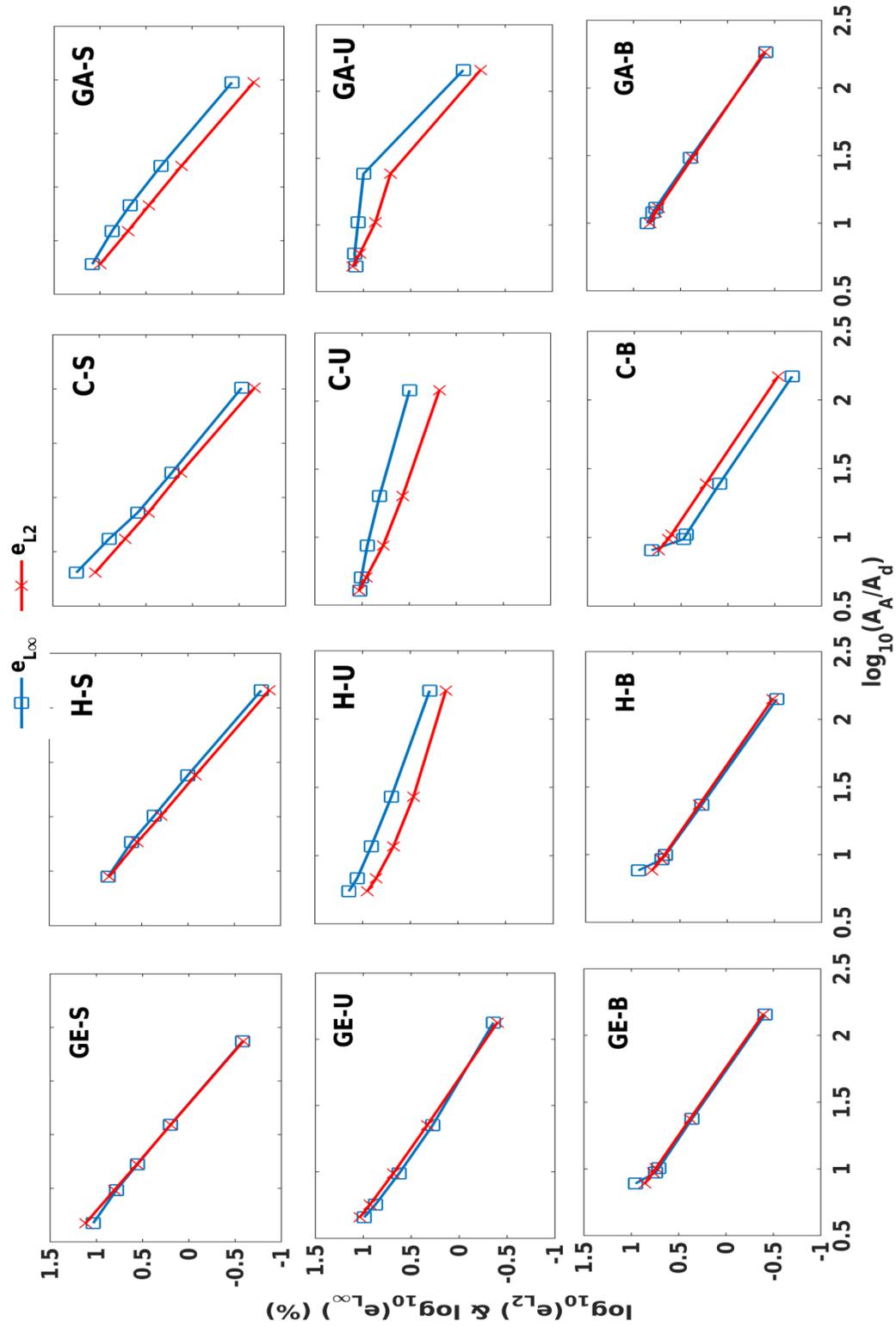
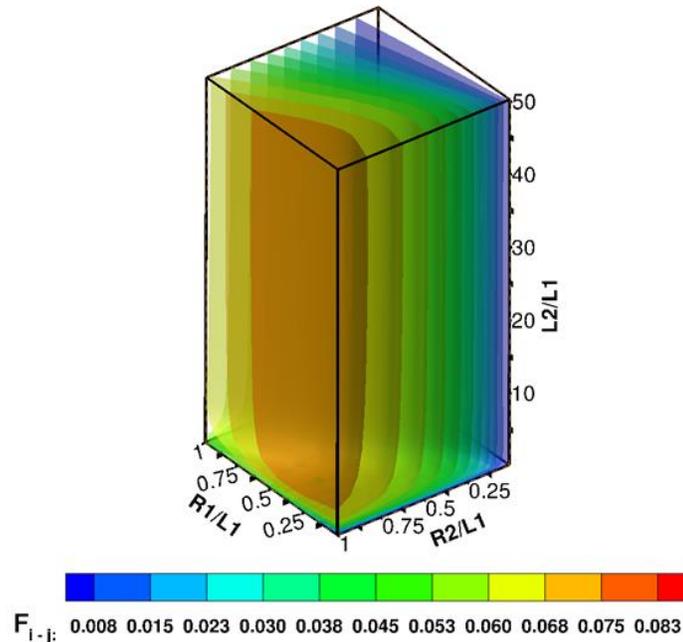


Figure 15.  $e_{L2}$  and  $e_{L\infty}$  (%) vs. Ratios of surface areas between attics and duct shapes of different set-ups.

### ***Result of Evaluation***

The assumption is valid when the ducts are in the intervals  $\frac{R_2}{L_1} \in \left[\frac{1}{200}, 0.5\right]$ ,  $\frac{R_1}{L_1} \in \left[\frac{1}{200}, 1\right]$  and  $\frac{L_2}{L_1} \in \left[\frac{1}{50}, 50\right]$ . In this situation, the view factors between two ducts inside an attic can be ignored since they have a limited effect on the overall view factor between a duct and attic surfaces. However, the assumption is may not valid when the ratio of  $\frac{R_2}{L_1} > 0.5$  and  $\frac{R_1}{L_1} \in \left[\frac{1}{200}, 1\right]$  and  $\frac{L_2}{L_1} \in \left[\frac{1}{50}, 50\right]$ . An efficient way to determine whether the view factors between two ducts in an attic negligible or not is to provide a fitting curve through all data presented in the volumetric plot. The equation is derived from the fitting curve and used to calculate view factors between two ducts. This process can be completed as future work as a further generalization to include the duct angle is needed. In this project, the view factors between two ducts will be considered negligible in order to simplify the calculation process in chapter 5.



**Figure 16. Isosurfaces representation of view factors controlled by three dimensionless parameters  $\left(\frac{R_1}{L_1}, \frac{R_2}{L_1}, \frac{L_2}{L_1}\right)$  in volumetric graph for ducts that are perpendicular to each other.**

## CHAPTER 5: COMPARISON BETWEEN THE NEW METHOD AND THE RP-717 METHOD

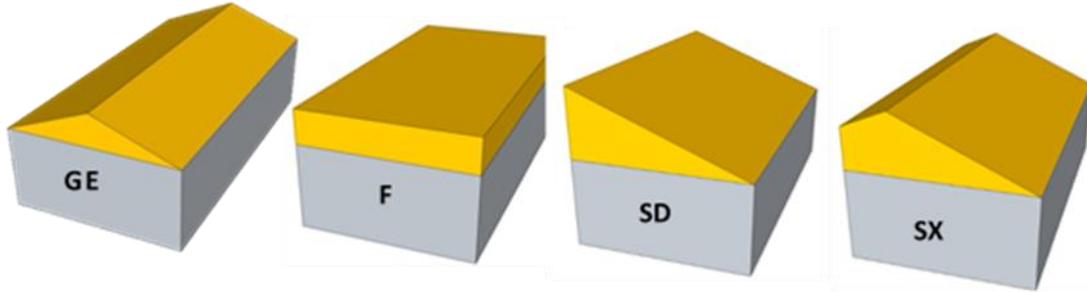
### 5.1 Brief Description of Method, Attic & Duct Base Configurations

After evaluating all view factor assumptions made in RP-717 and providing alternative methods to the inaccurate assumptions. It is necessary to evaluate the accuracy of the new method. Three different view factor matrices are be computed, 1) from View3D (considered the most accurate and benchmark), 2) from the assumptions in RP-717, and 3) from the new methods developed in chapter 4. The  $L_2$  relative error norm, eq. 25, is used to compare how close between the RP-717 assumptions and View3D and between the new method and View3D. This comparison step would show overall which method is more accurate compared to View3D.

$$e_{L2} [\%] = \sqrt{\frac{\sum_{i=1}^N \sum_{j=1}^N \left( F_{i \rightarrow j}^{(\text{method})} - F_{i \rightarrow j}^{(\text{View3D})} \right)^2}{\sum_{i=1}^N \sum_{j=1}^N \left( F_{i \rightarrow j}^{(\text{View3D})} \right)^2}} \times 100 \quad (25)$$

The different attic dimensions used in this section are constructed based on ASTM C1340 [15] as described in section 3.2. However, attic shapes are based on RP-717 such as gable ended, flat, shed, and saltbox attic geometries as shown in figure 16. It is desirable to see how the new approach (New  $F_{i \rightarrow j}$ ) improves view factors compared to RP-717 assumptions (Assumption  $F_{i \rightarrow j}$ ) on the shapes described in RP-717 project. Duct configurations used in this section are single (S), u-shape (U) & branch (B) as described in figure 4. Duct outer diameter for all segments is 1.16

ft. and circular cross section of duct used here which are similar to duct in ASTM C1340 [15]. The emittance factors for all attic and duct configurations are described in table 5.



**Figure 17. Attics shapes (left to right): gable-ended (GE), flat (F), shed (SD) & saltbox (SX).**

**Table 5. Four attic shapes combined with a duct configuration along with three different sets of emittance factors such as black body for all surfaces; roofs are made out of oriented strand board (OSB); attic floor is made out of fiberglass and duct is made out of fiberglass and duct is made out of either galvanized iron or aluminum.**

GE	S	Roof, Floor, Duct: black body
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Duct: galvanized iron (0.9)
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Duct: polished aluminum (0.05)
F	U	Roofs, Floor, Ducts: black body
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Ducts: galvanized iron (0.9)
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Ducts: polished aluminum (0.05)
SD	B	Roofs, Floor, Ducts: black body
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Ducts: galvanized iron (0.9)
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Ducts: polished aluminum (0.05)
SX	B	Roofs, Floor, Ducts: black body
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Ducts: galvanized iron (0.9)
		Roofs: OSB (0.91), Floor: fiberglass (0.75), Ducts: polished aluminum (0.05)

## 5.2 Evaluation Results between New Method and RP-717

Overall results in figure 17 show that the view factors of surfaces calculated by new methods are much better than RP-717 assumptions. The overall error is less than 10% for black body and two emittance cases, compared to up to 40% error from the RP-717 assumptions. As a result, the new method provides some improvements on the current view factor assumptions made in RP-717.

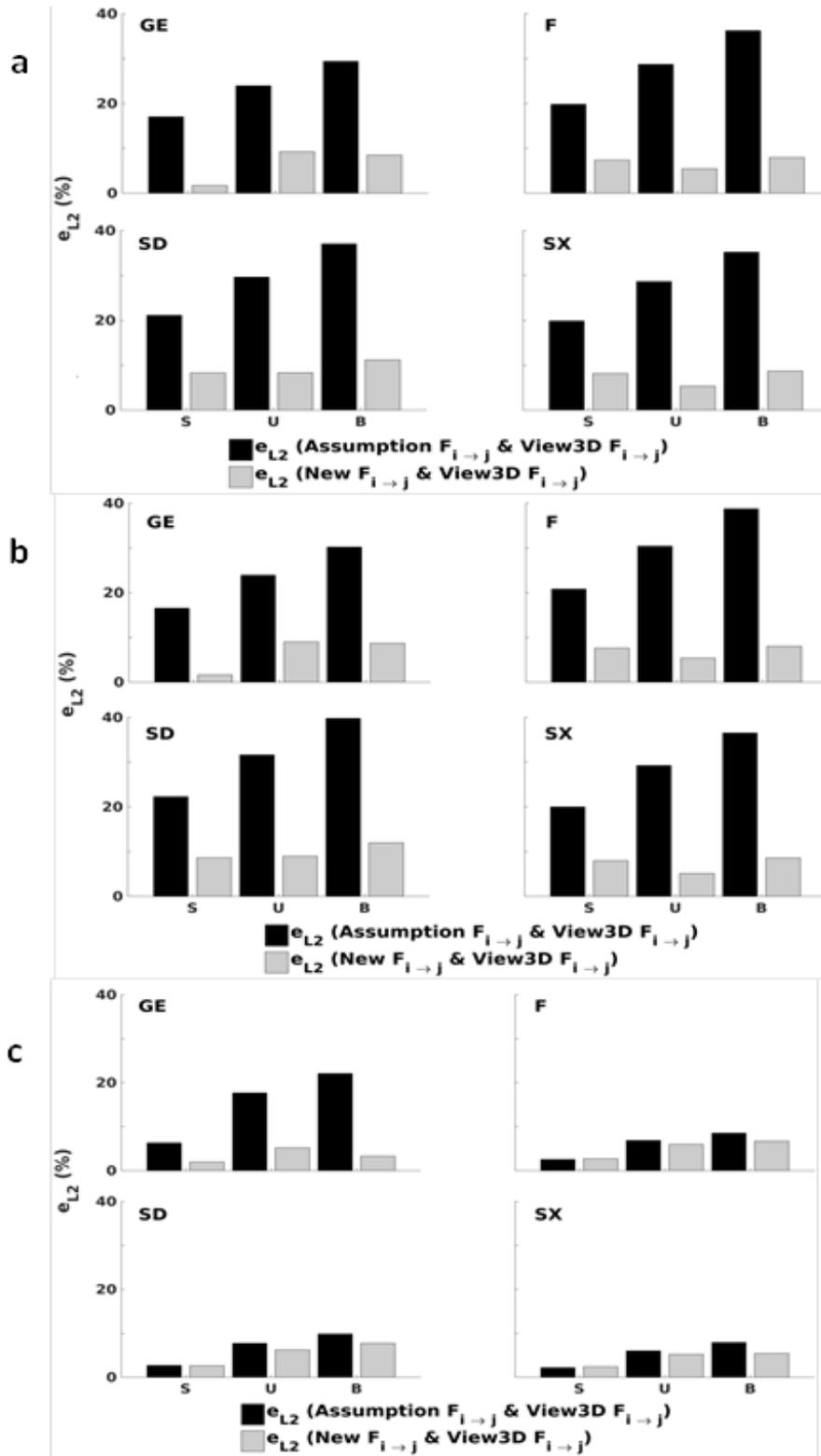


Figure 18. Errors EL2 between Assumption  $F_{i \rightarrow j}$  / New  $F_{i \rightarrow j}$  and View3D  $F_{i \rightarrow j}$ . From top to bottom: a) Black body, b) Fe and c) Al in different attic and duct geometries.

## CHAPTER 6: CONCLUSIONS & FUTURE WORK

### 6.1 Conclusions

A set of assumptions made in RP 717 is very useful in evaluating air duct view factors in an attic enclosure. However, there has been minimum investigation on those assumptions. The aim of this project is to evaluate the assumptions in order to identify which assumptions can be used to approximately calculate view factors of ducts in attics and suggest alternative methods to inaccurate assumptions. View3D simulation software and a list of attic and duct configurations are used in this research to benchmark duct view factor assumptions. The simulation results show the following:

- View factor from any duct to attic floor is 0.5 and the presence of ducts reduce the view factors from the attic floor to other attic surfaces by a constant factor can be used to approximately evaluate view factors of surfaces and ducts in an attic.
- If ratio of surface areas between attic and duct shape is bigger than 15, then the assumption, the presence of ducts modifies only those view factors that involve the attic floor can be used. At this ratio, ducts are small enough that they can be ignored in an enclosure. In contrast, if the ratio is less than 15 then the assumption cannot be used because surface area of ducts affects view factors of other attic surfaces. Thus ducts cannot be ignored. An alternative suggestion in that case is to use View3D to run simulation to calculate view factors among attic surfaces.
- Duct at different location in attic have different view factors to the different attic surfaces. Therefore, the assumption that the view factor from a duct to a

particular attic surface is the same for all duct segments cannot be used. The values calculated by using the assumption are not the same as results obtained from View3D simulations. An alternative suggestion is to introduce the product of two ratios into the original assumption method. One is ratio of centroid distances between duct and enclosure surfaces. The other is an area ratio. Alternative method shows better view factors than assumption view factors.

- The view factor of two ducts connected one end ( $\theta = 90^\circ$ ) might be larger than 0.05. Hence the assumption, ducts are small enough that view factors between any two ducts may be considered negligible compared with view factors between the ducts and the attic surfaces might not be correct in some cases. The future work is to construct a function that can be interpolated between simulation points. It can be used to determine view factors between two ducts in an attic enclosure.
- Finally, the comparison is made between the new method which has valid assumptions along with alternative methods and the RP-717 method which has all original assumptions. The results show that view factors matrices obtained by new method are more accurate than those obtained by RP-717 original assumptions. Therefore, they are useful in determining view factors of ducts in residential and light commercial buildings.

The verification process provides useful information about validity air duct view factor assumptions. This project helps engineers and researchers realize advantages and disadvantages of each assumption in computing air duct view factors. As a result, energy

evaluation process of air ducts in an attic space can be appropriately evaluated and energy consequences of having air ducts in an attic space are minimized.

## **6.2 Future Work**

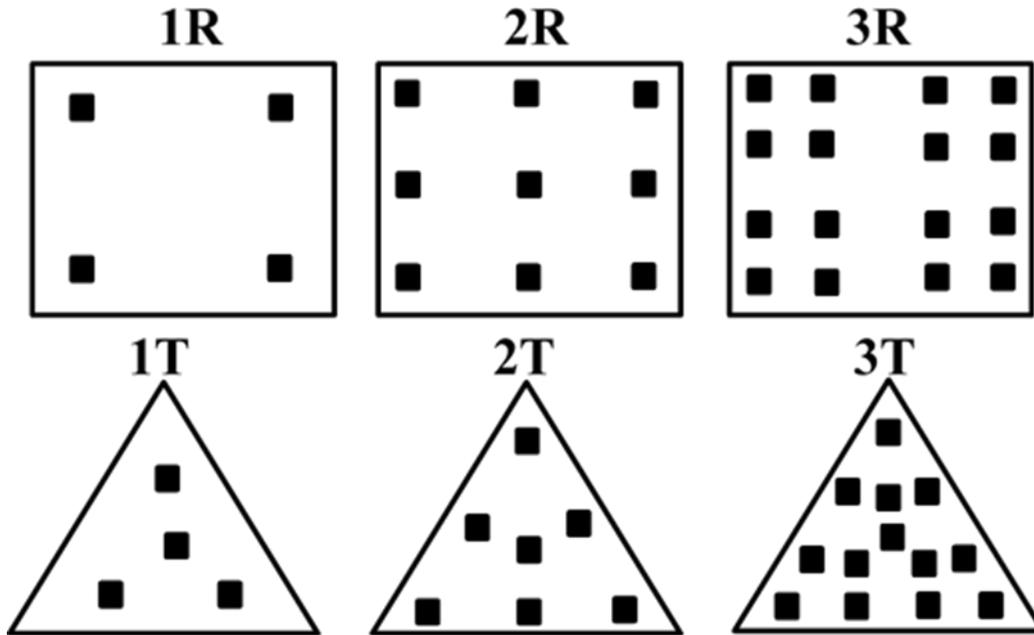
There are some improvements that need to be made in order to increase the accuracy and efficiency of view factor calculations. First, it may be necessary to make improvements on the projected areas of duct on enclosure surfaces. Hence, better view factors between a duct and any attic surfaces can be computed correctly. Second, generalization of duct to duct view factors is also needed to include acute angles, and a fitting function to quickly and easily produce the view factors is needed. Third, square duct is also a common duct shapes in light commercial buildings. Therefore, it is good to investigate view factors of this shape in an attic by using valid assumptions and alternative methods done in this work. The sensitivity to thermal loads based on different view factors calculation algorithms also needs to be investigated. Finally, all valid assumptions and alternative methods could be developed as algorithms and functions in C++ for future implementation in other frameworks like FATM, EnergyPlus, or ESP-r.

## **APPENDIX OPERATION OF VIEW3D**

In this appendix, the general description of numerical methods implemented in View3D is shown. They are used in three different situations that View3D solves for view factors of surfaces in an enclosure, such as unobstructed, obstructed and emitting view factors.

### **A1. Unobstructed View Factor**

View 3D uses three different methods to evaluate unobstructed view factor: double area integration, double line integration [24] and Mitalas and Stephenson (MS) method [24]. In order to improve the accuracy of both double line integrals and MS method, Gaussian integration is used. The method divides each edge of a surface into elements of different lengths. Then the view factor function is evaluated based on selected Gaussian points in those elements [24]. The program increases the number of Gauss points until view factors converges. An example of selected Gaussian points on triangle and rectangle elements is shown in figure 18. More Gaussian points are taken on surface of the rectangle (1R to 3R) until view factor of two surfaces reach convergent value. Same principle is implemented on surface of triangle (1T to 3T)

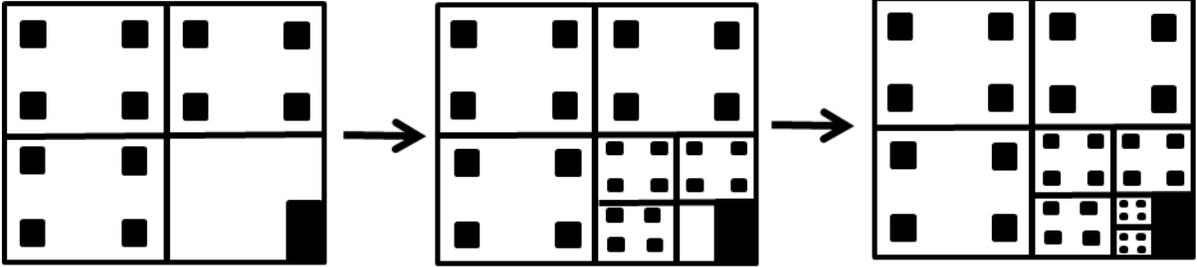


**Figure 19. Selected Gaussian points on triangle and rectangle elements.**

## **A2. Obstructed View Factor**

View3D software examines all surfaces in an enclosure to determine obstruction surfaces. There are several algorithm tests used in the software to determine obstruction surfaces. They are self-obstruction, cylinder radius test, centroid project test and obstructing effect [16]. In order to compute correctly view factor from a surface to a portion of another surface that is not obstructed, View3D uses adaptive integration. Basically, the program keeps dividing the unobstructed portion of the surface and applying Gaussian quadrature to compute view factor of two surfaces until the difference between view factor of current and previous division less than specific tolerance  $\beta$ , eq. A1. An example of adaptive integration is shown in figure 19. Adaptive integration is crucial to control the number of divisions because few divisions lead to less precise view factors and too many will potentially waste a lot of computational time [16].

$$|AF^{[k+1]} - AF^{[k]}| < \beta A_{\min} \quad (A1)$$



**Figure 20. Adaptive integration method is implemented on rectangle element.**

As seen in figure 19, there is an obstruction represented as a black rectangle at the bottom - right corner on the surface. The program first divides the rectangle into smaller elements and use unobstructed method as discussed previously to calculate view factor of two surfaces (1), excluding obstruction surface. If convergent condition is not met, then more divisions and view factors are processed (2 to 3) until eq. A1 is satisfied.

### **A3. Modifying black body view factors to diffusely emitting view factors**

Besides calculating unobstructed and obstructed view factors of blackbody in an enclosure. View3D can also handle diffusely emitting surfaces. View3D solves view factors of all surfaces in an enclosure first as a blackbody. After that, the software will use an algorithm that has been developed by H.C. Hottel and A. F. Sarofim [17] in order to take into account emittance factors of enclosure surfaces, eq. A2. Basically it solves system of equations  $[\mathbf{T}]$   $[\mathbf{W}] = [\mathbf{E}]$  for leaving flux density  $[\mathbf{W}]$  of each surface, eq. A3. Then it makes some adjustments for  $[\mathbf{W}]$  to solve for diffusely emitting view factors, eq. A5.

$$\frac{-\epsilon_j A_j}{\rho_j} E_j = \sum_i W_i \left( \overline{S_i S_j} - \frac{\delta_{ij} A_j}{\rho_j} \right) \quad (A2)$$

Equation A3 represents leaving flux of all surfaces in an enclosure. The leaving flux density of each surface depends on areas, black body view factors, emissivity and reflectance of its own and other surfaces. Equation A2 can be written as system of equations:

$$\underbrace{\begin{bmatrix} \overline{s_1 s_1} - \frac{A_1}{\rho_1} & \overline{s_1 s_2} & \overline{s_1 s_3} & \dots \\ \overline{s_1 s_2} & \overline{s_2 s_2} - \frac{A_2}{\rho_2} & \overline{s_2 s_3} & \dots \\ \overline{s_1 s_3} & \overline{s_2 s_3} & \overline{s_3 s_3} - \frac{A_3}{\rho_3} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}}_{\text{Transfer matrix}} \times \underbrace{\begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ \vdots \end{bmatrix}}_{\text{Response Vector}} = \underbrace{\begin{bmatrix} \frac{-\epsilon_1 A_1}{\rho_1} E_1 \\ \frac{-\epsilon_2 A_2}{\rho_2} E_2 \\ \frac{-\epsilon_3 A_3}{\rho_3} E_3 \\ \vdots \end{bmatrix}}_{\text{Excitation Vector}} \quad (\text{A3})$$

$$\overline{s_i s_j} = A_i F_{i \rightarrow j} \quad (\text{A4})$$

In order to find leaving flux densities (response vector) of each surface, the transfer matrix first is filled with all direct interchange area, eq. A4 of black body and ratios of surface area and reflectance. Then the excitation vector is filled based on Kronecker delta operation,  $\delta_{ij} = 0$  if  $i = j$  and  $\delta_{ij} = 1$  if  $i \neq j$ .  $E_j$  is treated as a constant factor so it is assumed to be 1 during the calculation process. Finally the response vector is solved for each surface of an enclosure. For an example, an enclosure is made out of 3 surfaces 1, 2 and 3 and each has different emit factors. The leaving flux density of each surface must include emittance of itself and the rest. Hence, after implementing system of equations A3, three sets of leaving flux densities are obtained for surface 1, 2 and 3 such as  ${}_1W_1, {}_1W_2, {}_1W_3$ ;  ${}_2W_1, {}_2W_2, {}_2W_3$  and  ${}_3W_1, {}_3W_2, {}_3W_3$ . They all have the form  ${}_iW_j$  and it means leaving flux density is on surface  $j$  when surface  $i$  emits.

After having all flux densities of each surface, they will be used as inputs for post-processing step, eq. A5 in order to produce new view factors that include emittance factors.

$$\overline{S_i S_j} = \frac{-\epsilon_j A_j}{\rho_j} \left( \frac{W_j}{E_i} - \delta_{ij} \epsilon_i \right) = A_i F_{i-j} \quad (A5)$$

First, equation A5 is filled with emittance factors, reflectance and leaving flux densities following Kronecker delta operation. Next, the total-exchange area,  $\overline{S_i S_j}$  can be solved. Finally, using it divided by surface area,  $A_i$  to solve for new view factor,  $F_{i-j}$ .

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